Coherent electron Cooling:
What can be Analyzed, Estimated and Simulated

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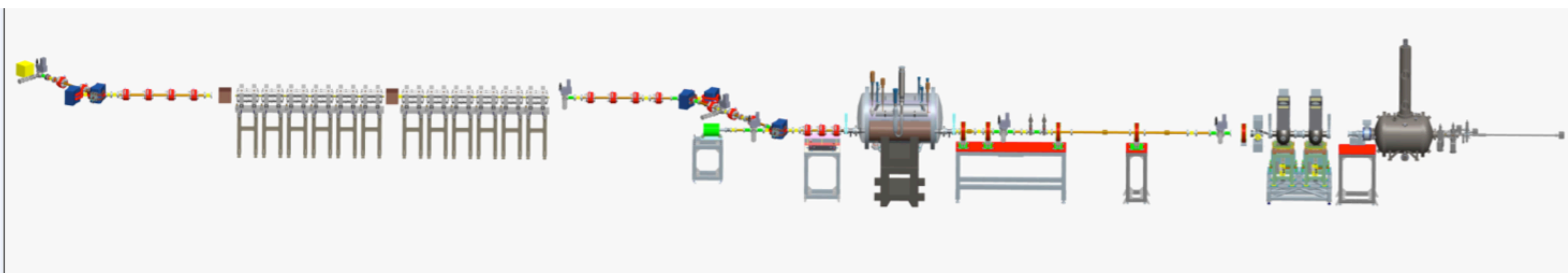

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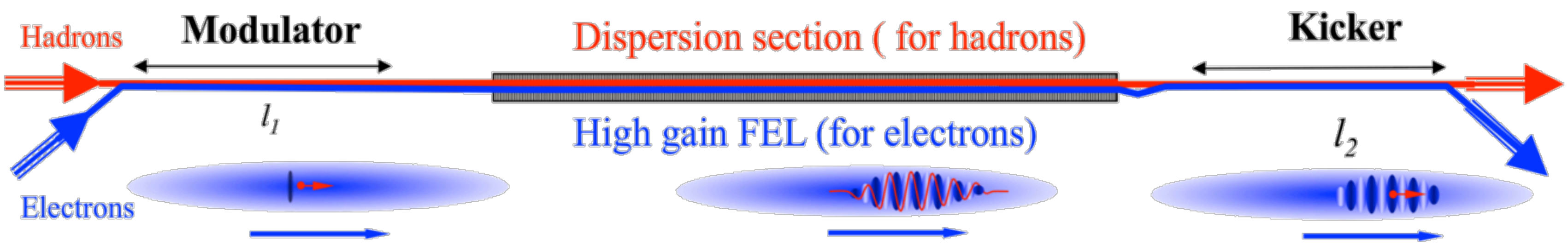
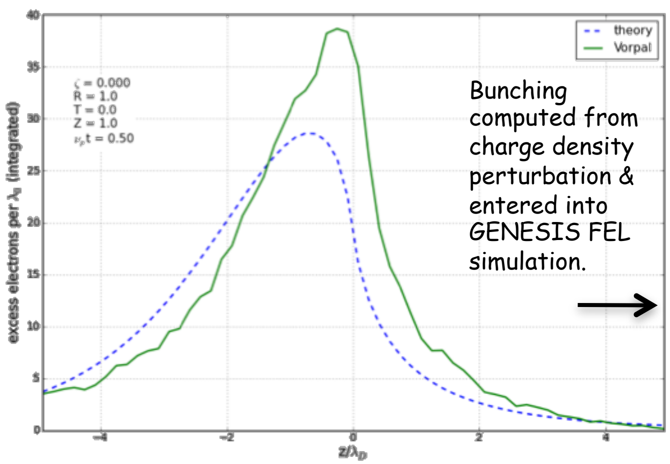
Outline

- Expected electron beam parameters from beam dynamics simulation
- Tools to analyze one pass CeC process
  - Modulation
  - FEL amplification
  - Kicker
- Tools to predict evolution of ion beam under cooling
  - Solving Fokker-Planck equation
  - Macro-particle tracking
- Summary

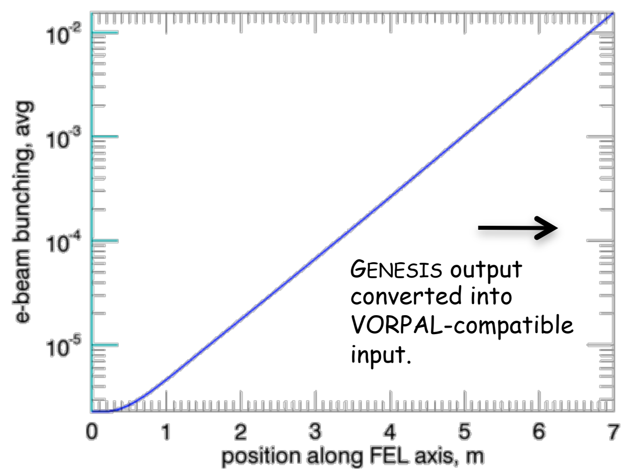
Our PoP is based on an economic version of CeC: it limits strength of the wiggler a_w to about 0.5 **but it is very cost effective**



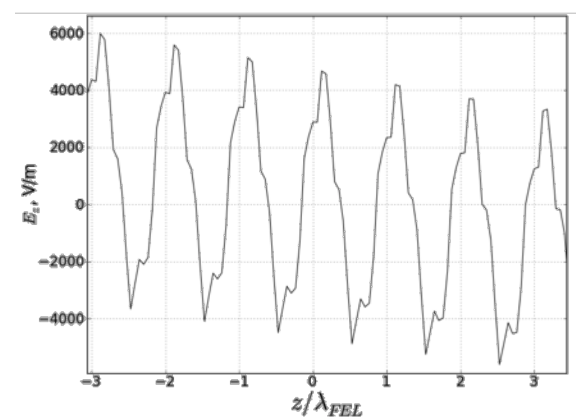
Param.'s from 40 GeV proof-of-principle exp. at BNL



VORPAL 3D δf PIC computation of e- density perturbation near Au^{+79} ion (green) vs. idealized theory (blue). On Cray XE6 cluster at NERSC.



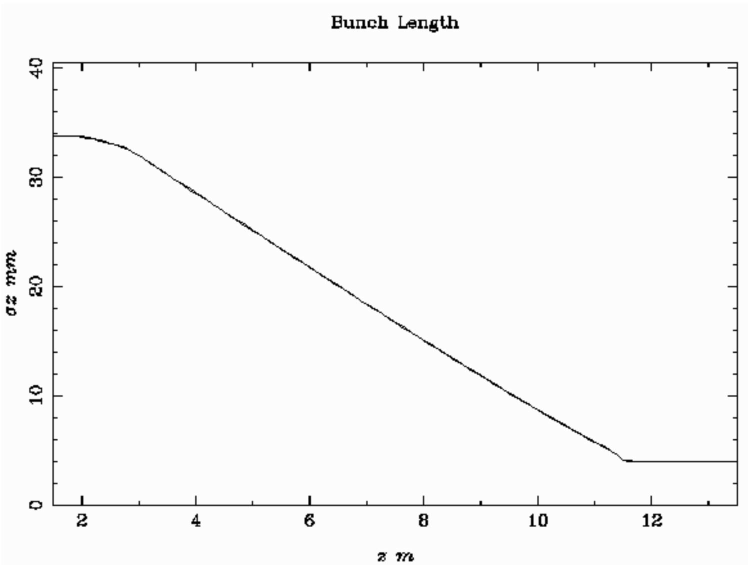
GENESIS parallel computation of electron beam bunching in free electron laser (FEL) shows amplification of modulator signal.

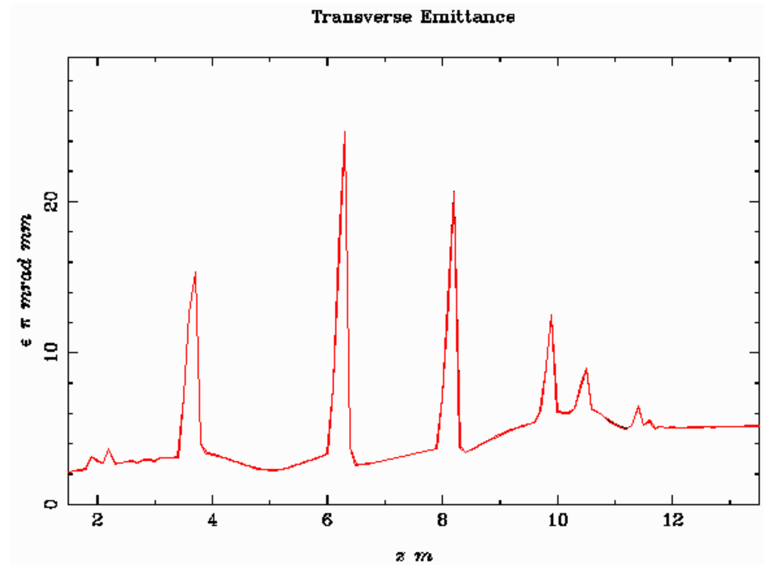


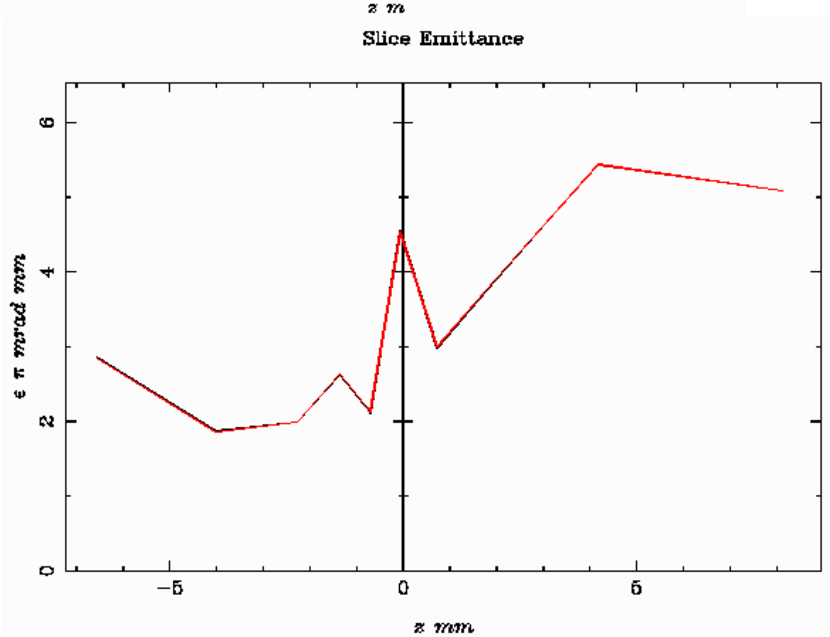
VORPAL prediction of the coherent kicker electric field E_k due to e-density perturbation from modulator, amplified in the FEL.

Simulations by Tech-X

Electron beam dynamics simulation







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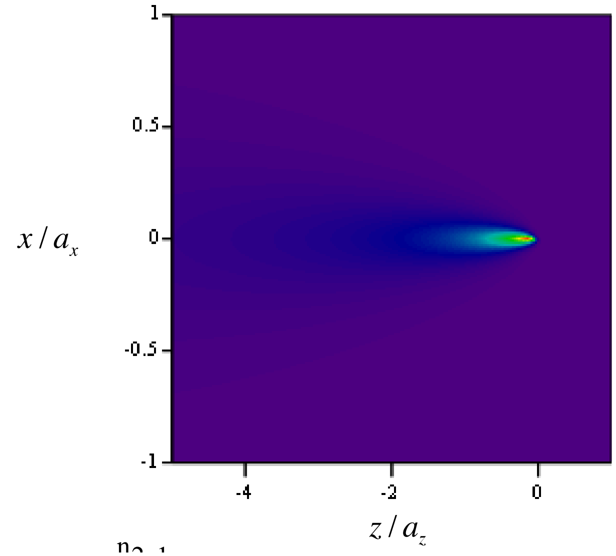
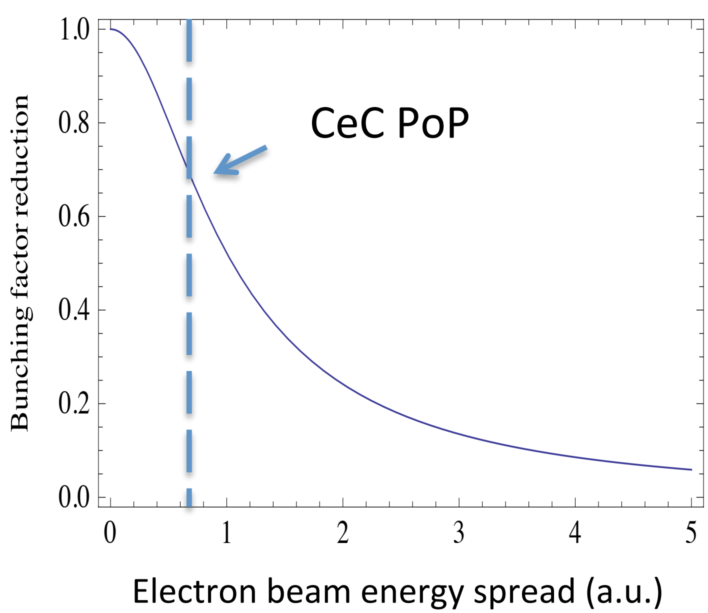
- For 2nC bunch charge, Astra simulation shows the following electron beam parameters at the exit of the linac:
  rms transvesrse emittance: 5 mm.mrad,
  rms bunch length: 4 mm
  energy spread: ~10^3

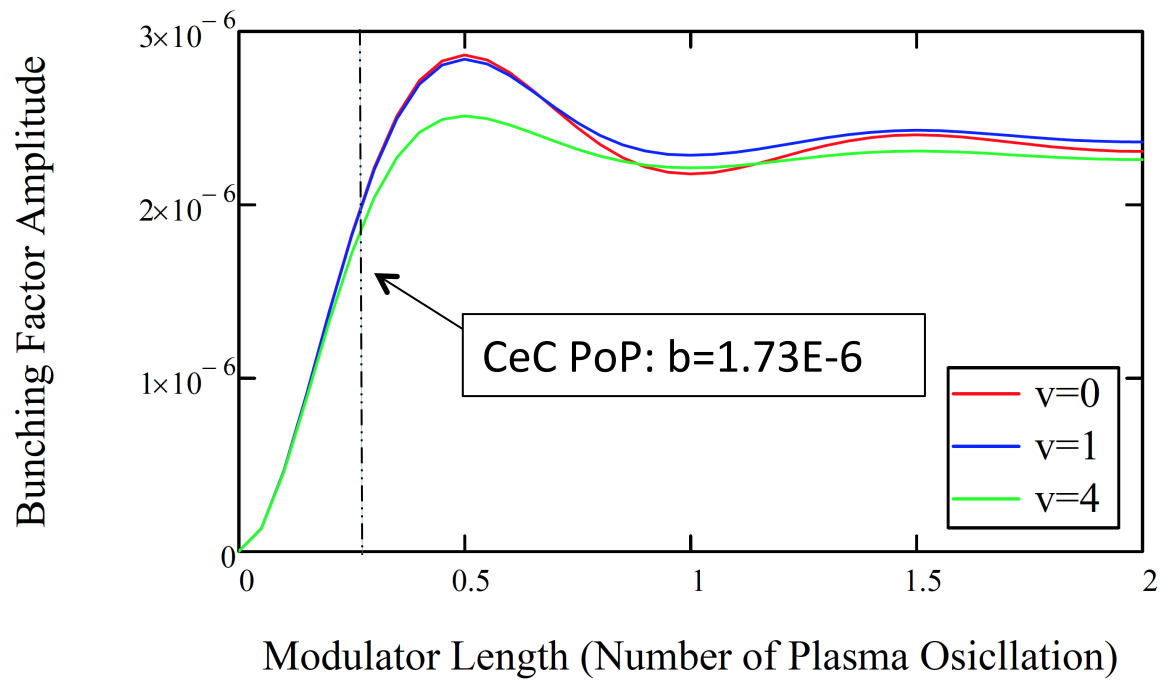
Modulation I:



- Density modulation of electrons due to Debye shielding

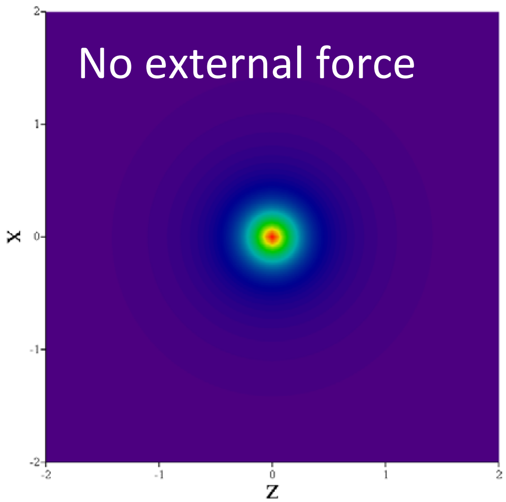
$$\tilde{n}_1(\vec{x},t) = \frac{Z_i}{\pi^2 a_x a_y a_z}\int_0^{\omega_p t} \frac{\tau\sin\tau\cdot d\tau}{\left[\tau^2 + \left(\frac{x}{a_x}+\frac{v_{0,x}}{\beta_x}\tau\right)^2 + \left(\frac{y}{a_y}+\frac{v_{0,y}}{\beta_y}\tau\right)^2 + \left(\frac{z}{a_z}+\frac{v_{0,z}}{\beta_z}\tau\right)^2\right]^2}$$

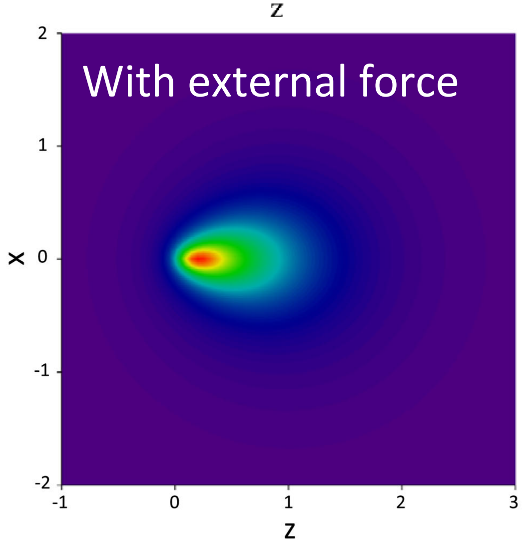


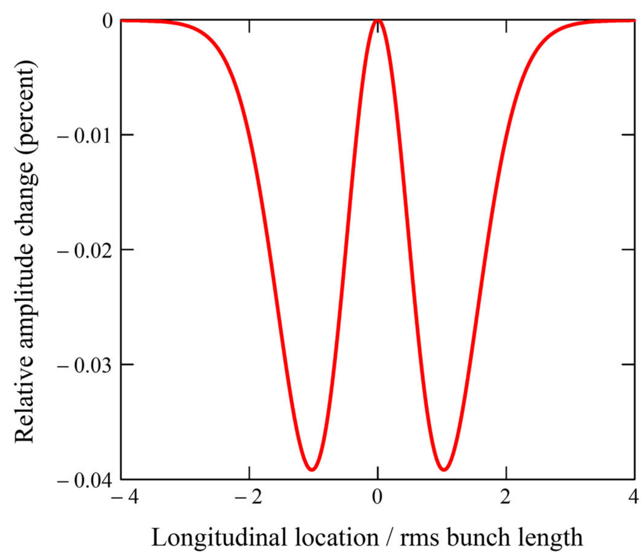


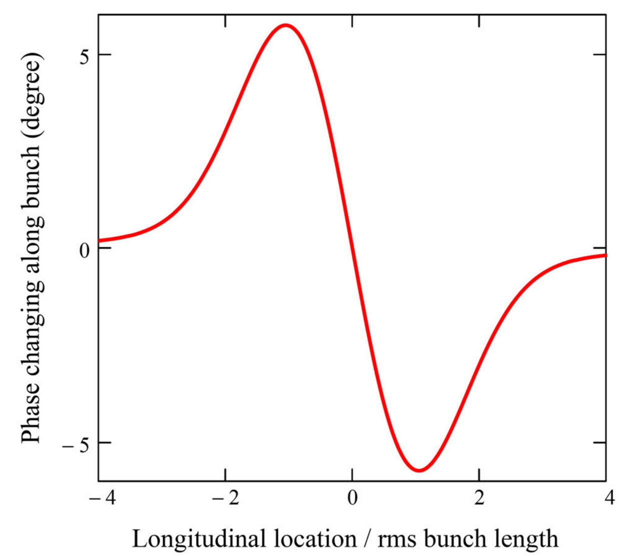
Improved Model for CeC Modulation:
Influences from long-range field

$$n_1(\vec{x},t)=\frac{Z_i}{\pi^2 r_x r_y r_z}\int_0^{\omega_p t}\psi\sin\psi\left[\psi^2+\left(\bar{x}-\bar{a}_x\psi\left(\omega_p t-\frac{\psi}{2}\right)+\bar{v}_{0,x}\psi\right)^2+\left(\bar{y}-\bar{a}_y\psi\left(\omega_p t-\frac{\psi}{2}\right)+\bar{v}_{0,y}\psi\right)^2+\left(\bar{z}-\bar{a}_z\psi\left(\omega_p t-\frac{\psi}{2}\right)+\bar{v}_{0,z}\psi\right)^2\right]^{-2}d\psi$$





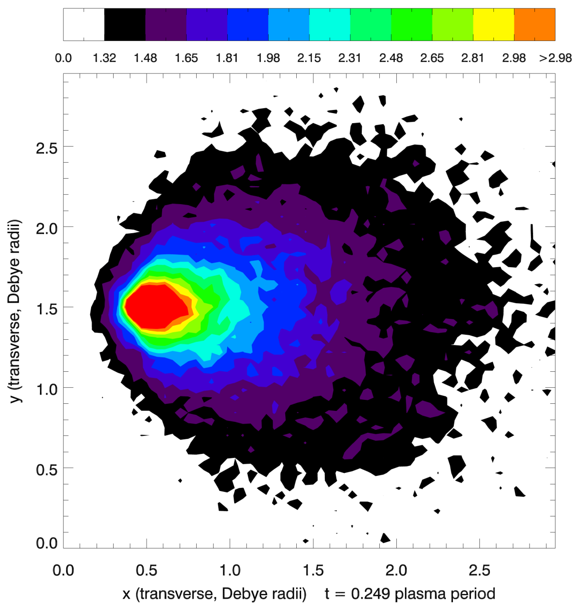


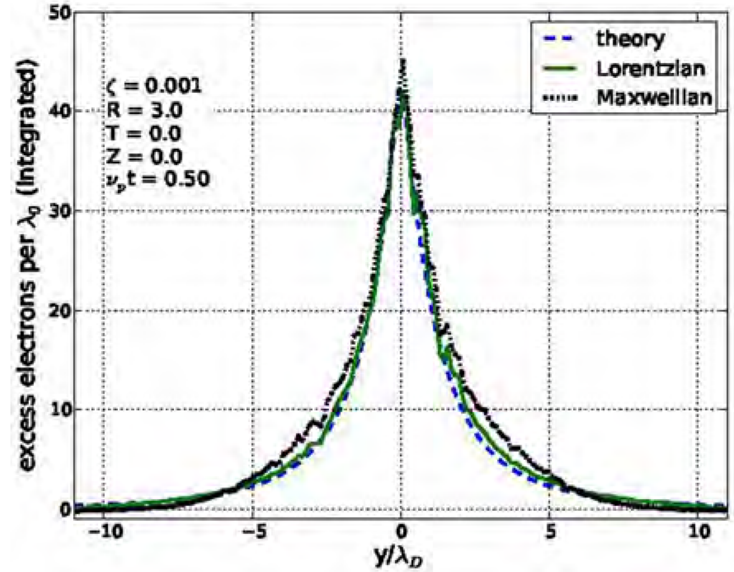


- The improved model allows for investigating the influences of long-range field on the CeC modulation process: ion shielding;
- For CeC PoP, the reduction of modulation due to longitudinal space charge field is small.

Simulations of modulator: uniform beam

- Simulation of the CeC modulator agree well with the analytical results.





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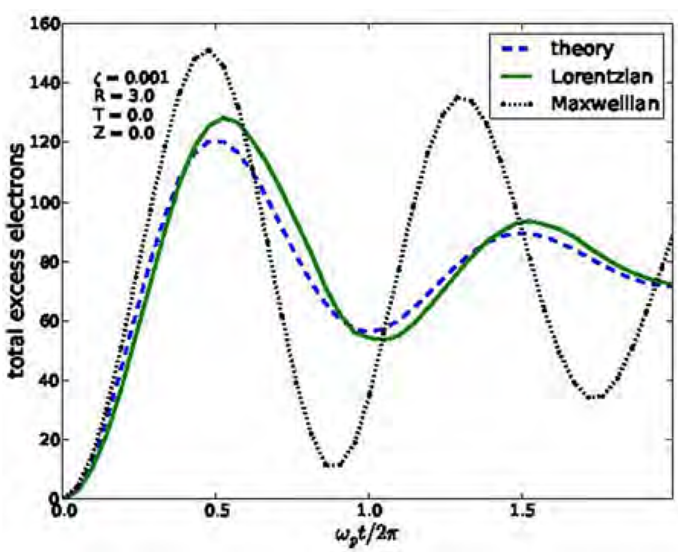

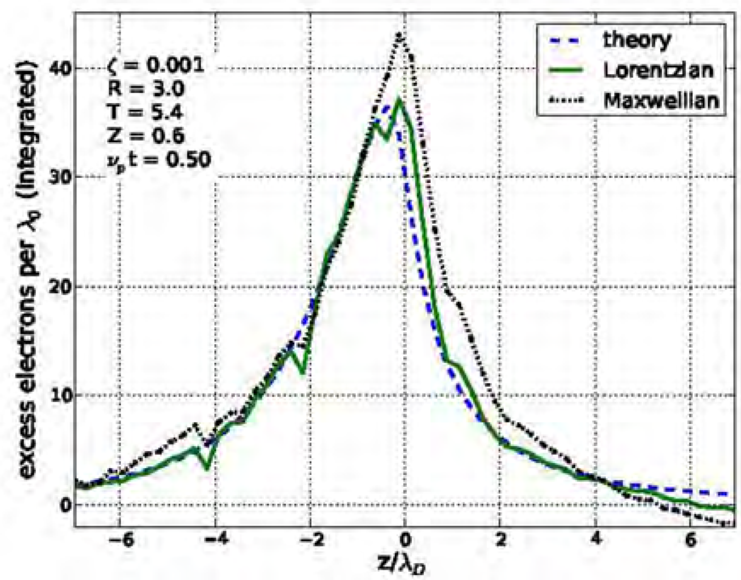

Modulator simulation with finite beam...

What has been done...

- Modulator simulation with uniform/continous external focusing

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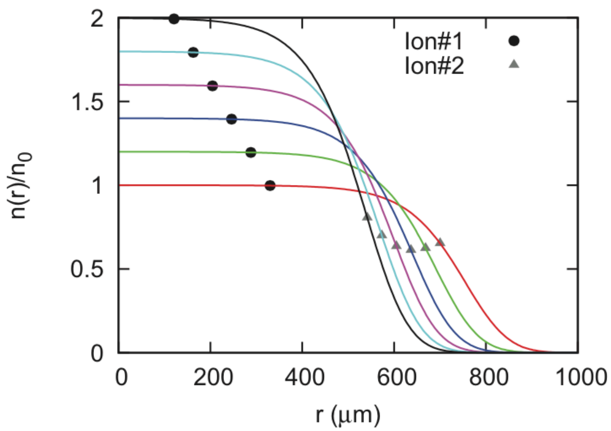

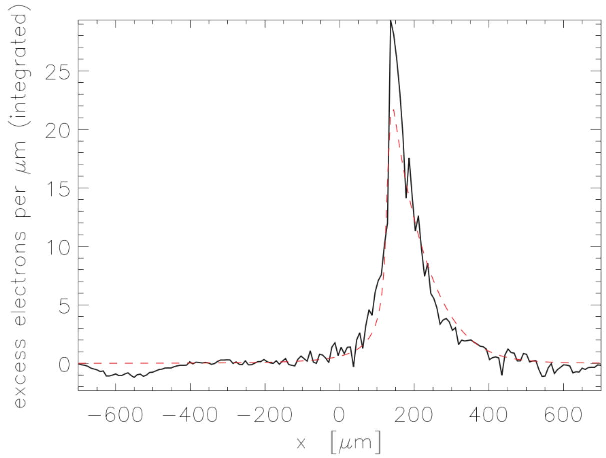

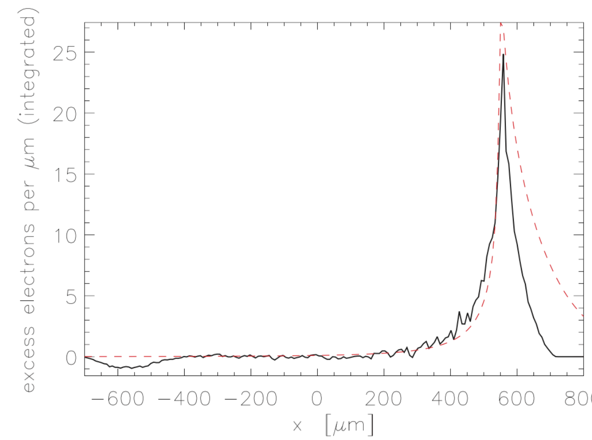

The simulation has been done by Tech-X. However, funds is limited and we presently do not have funds to support this direction at Tech-X.

- Solving Vlasov equation (with unperturbed trajectory approach):

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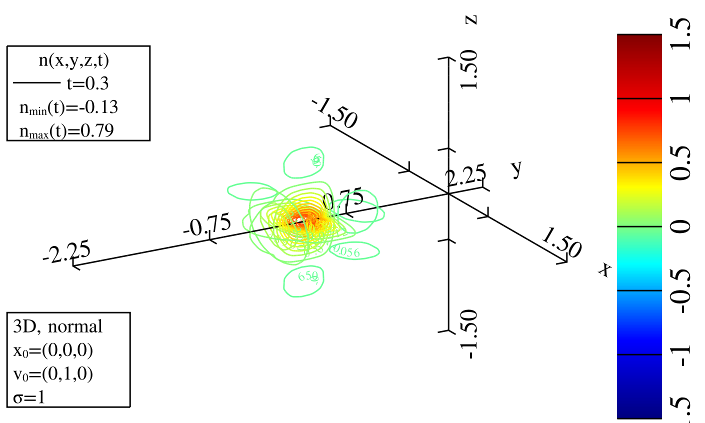
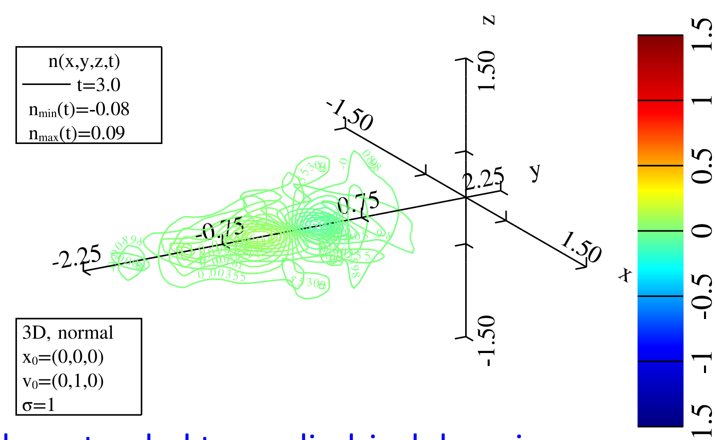


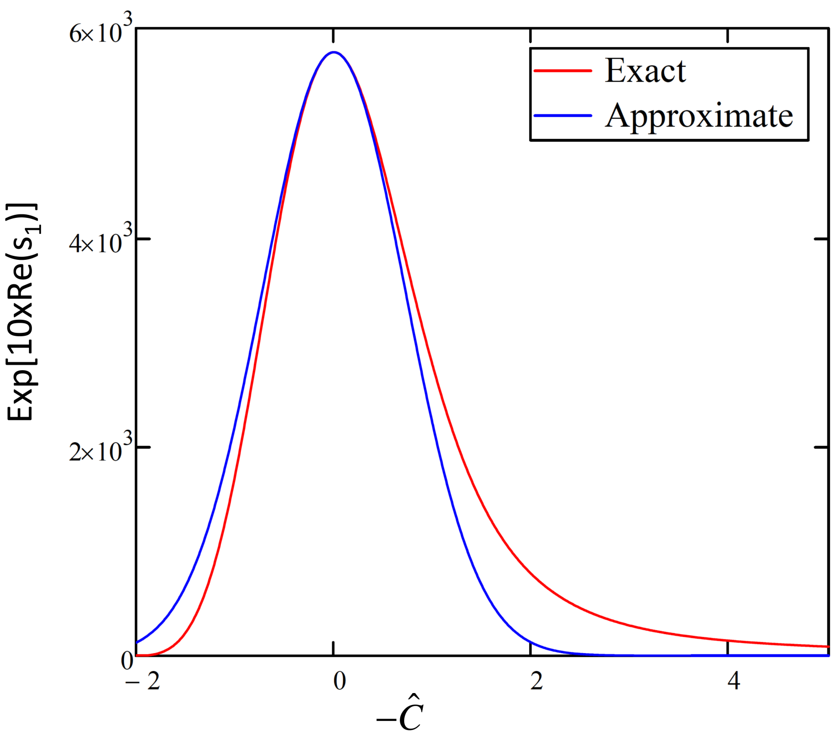


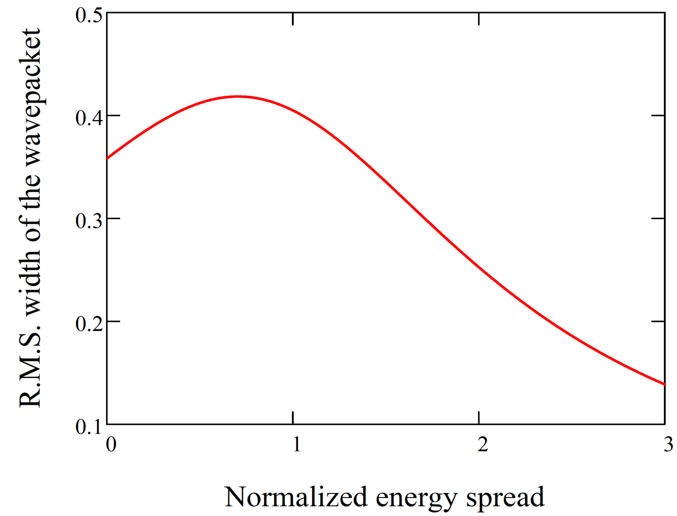
The current solution is for a spherical domain, which needs to be extended to a cylindrical domain.

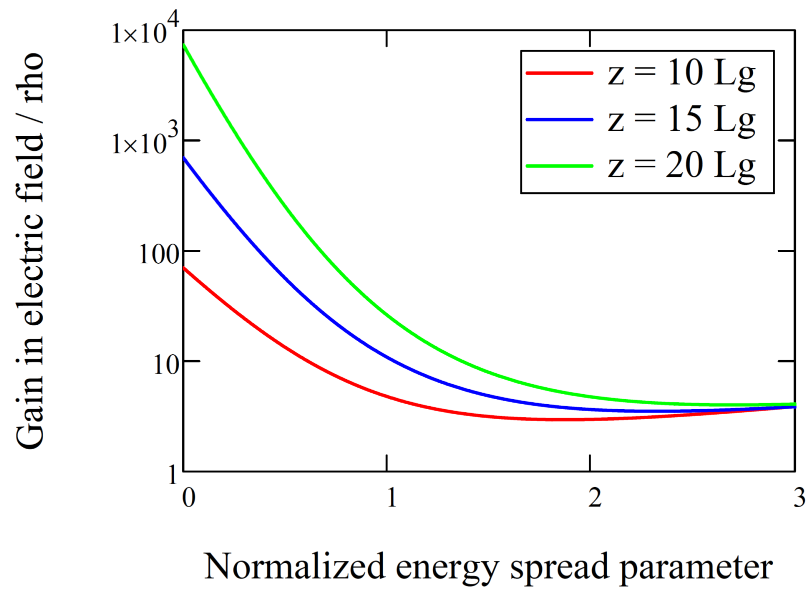
FEL amplifier (1D theory)

In high gain limit, we can expand the FEL dispersion relation to quadratic order in frequency and obtain the electric field due to electron density wave-packet:

$$E_{1D}(\tilde{z}) \approx E_p e^{-\frac{\tilde{z}^2}{2\sigma_{\tilde{z},rms}^2}} \sin(k_0\tilde{z} - \varphi_0)$$

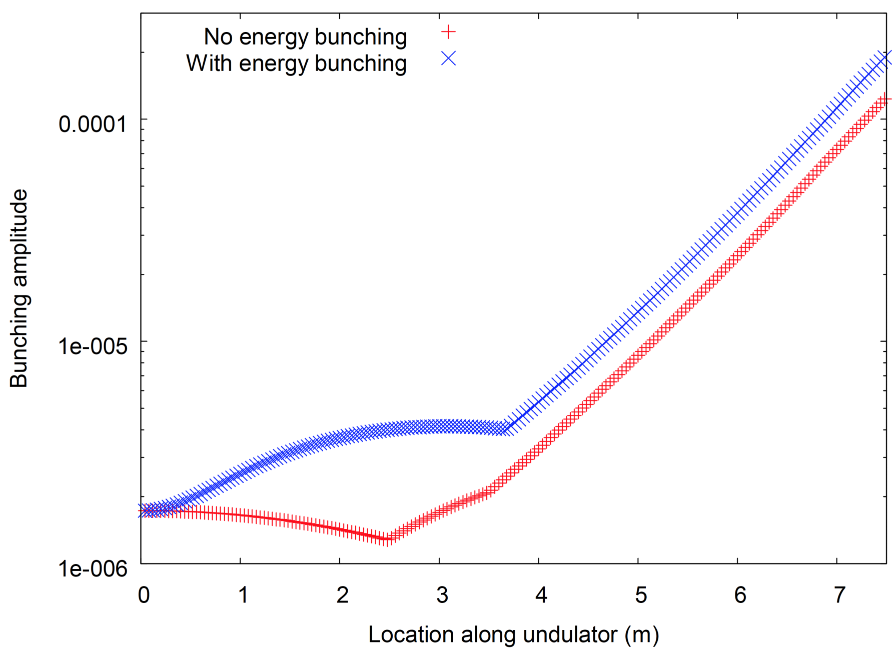


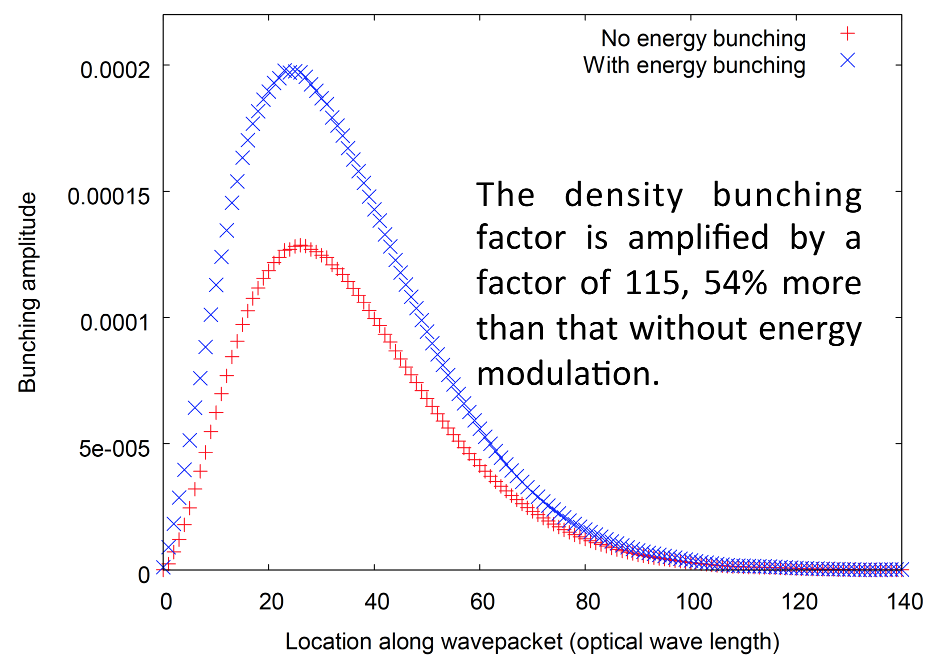




FEL amplifier simulation I:

Without shot noise from electrons (quiet start):





The density bunching factor is amplified by a factor of 115, 54% more than that without energy modulation.

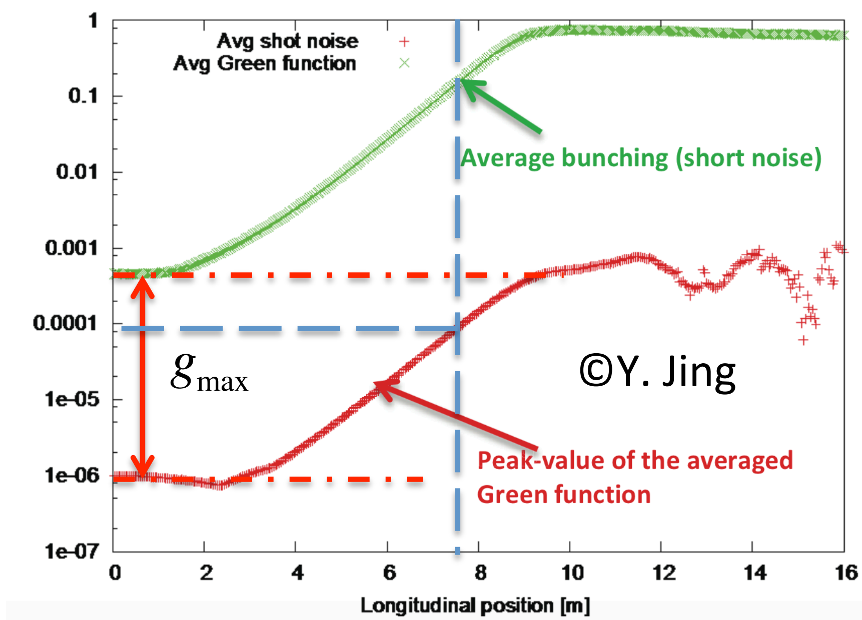
FEL amplifier simulation II:

With shot noise from electrons:

$$\left|\delta\hat{n}/n_0\right|_{\max} < 1 \Rightarrow \quad |g|_{\max} < \frac{\lambda_o}{2}\sqrt{\frac{I_e}{ecL_c}} \Rightarrow \quad g_{\max} \sim 72\cdot\sqrt{\frac{I_e[A]\cdot\lambda_o[\mu m]}{M_c}} = 429$$

$$M_c \equiv \frac{L_c}{\lambda_1} = \frac{1}{\lambda_1 g_{\max}^2}\int_{-\infty}^{\infty}\left|g(z)\right|^2 dz$$

- γ=21.8
- Peak current: 100 A
- Norm emittance 5 mm mrad
- RMS energy spread 1e-3
- λw=4 cm
- $a_w = 0.4$
- λo=12.7 um
- Mc = 35.8

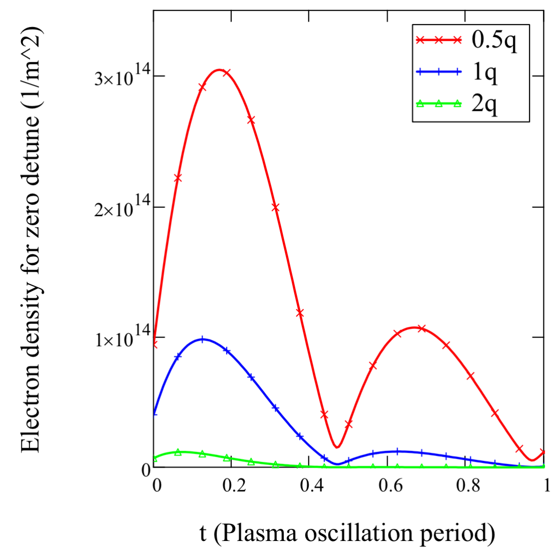


3D Genesis simulation shows that the maximal gain in bunching factor is 409, which agrees with our estimation.

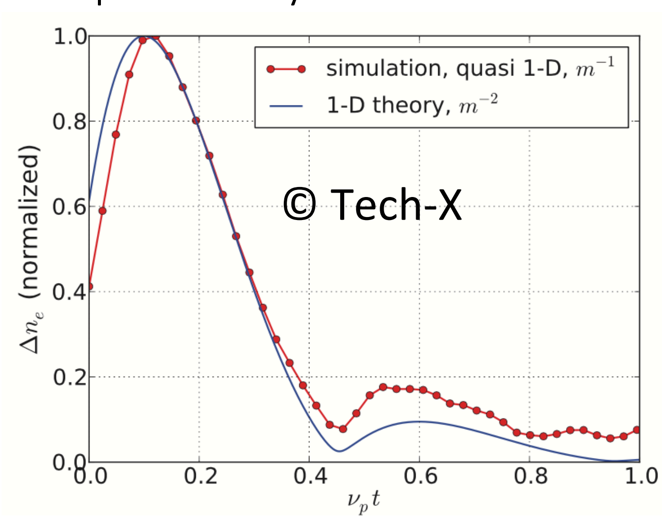
Kicker: 1D Model

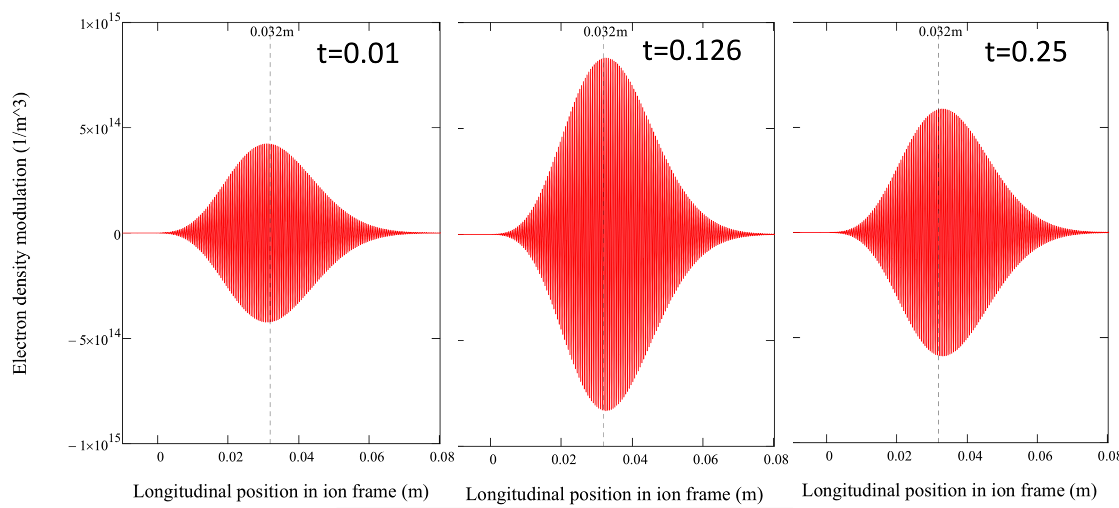
- For 1D FEL output with the following initial transverse perturbation,

$$\tilde{n}_1(k_z,t) = -Z_i \tilde{\Lambda}_{drive}(k_z) e^{ik_z v_{0z} t} e^{-|k_z|\sigma_{vz} t} \left[\cos(\omega_p t) + \frac{(\lambda_1 + i\hat{C})\beta_z c \gamma_z \Gamma - i k_z v_{0z} + |k_z| \sigma_{vz}}{\omega_p} \sin(\omega_p t) \right]$$



- The analytical result has been used to benchmark simulation studies performed by Tech-X.





Field Reduction due to Finite Transverse Modulation Size

$$\rho(\vec{r}) = \rho_o(r)\cdot\cos(kz);$$

$$\Delta\varphi = -4\pi\rho \Rightarrow \varphi(\vec{r}) = \varphi_o(r)\cdot\cos(kz);$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi_o}{dr}\right) - k^2\varphi_o = 4\pi\rho_o(r)$$

$$\rho(r) = \rho(0)\cdot g(r/\sigma)$$

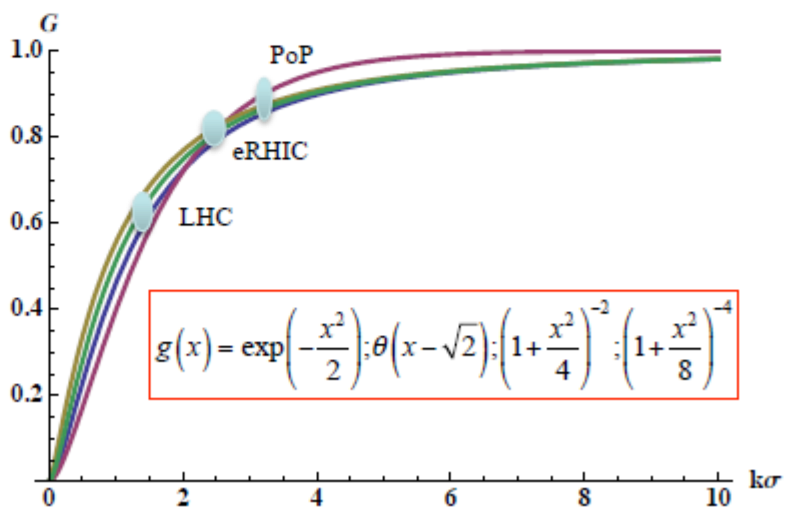
$$E_{zo}(r=0) \propto -\frac{4\pi\tilde{q}}{\sigma^2}G(k_{cm}\sigma)$$

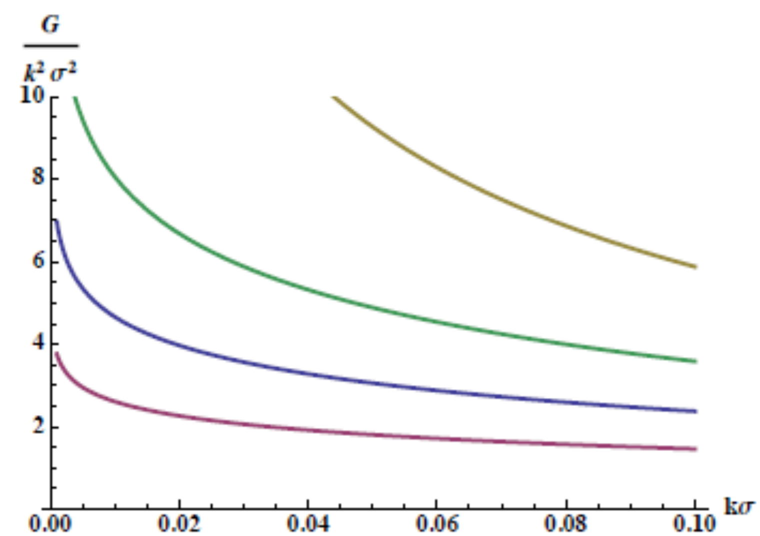
$$\varphi(\vec{r}) = -4\pi\cos(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

$$E_z = -\frac{\partial\varphi}{\partial z} = -4\pi k\sin(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

$$E_r = -\frac{\partial\varphi}{\partial r} = 4\pi k\cos(kz)\left\{I_1(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi - K_1(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

$$\boxed{k_{cm}\sigma_\perp = \frac{k_o}{\gamma_o}\sqrt{\frac{\beta_\perp\varepsilon_{n\perp}}{\gamma_o}} = \sqrt{\gamma_o}\sqrt{\beta_\perp\varepsilon_{n\perp}}\,\frac{k_w}{2(1+a_w^2)}}$$





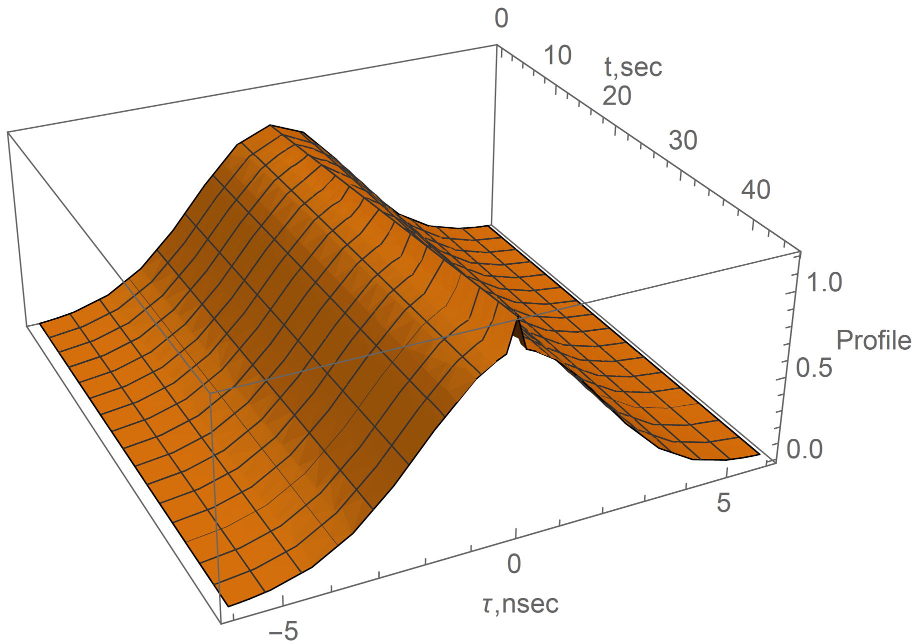
The results reproduces what previously derived by G. Stupakov

Evolution of ion beam under cooling:

Solving Fokker-Planck equation

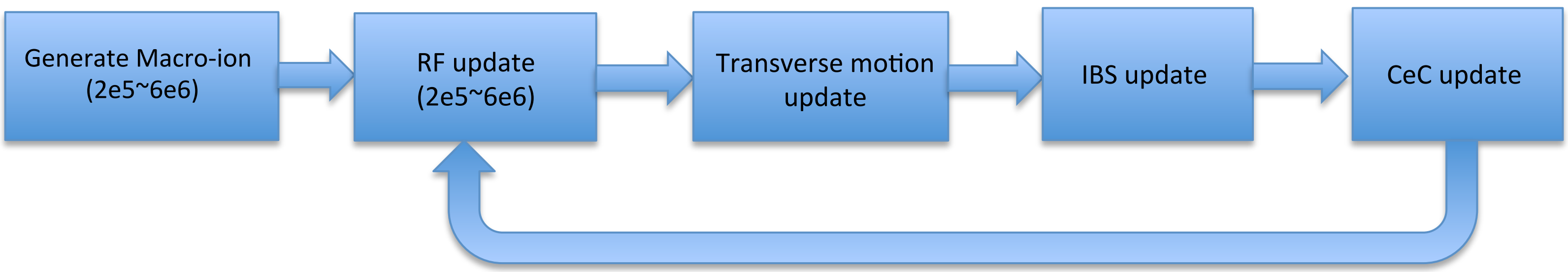
$$\frac{\partial F(I,t)}{\partial t}-\frac{\partial}{\partial I}\left(\zeta(I)\cdot I\cdot F(I,t)\right)-\frac{\partial}{\partial I}\left(D(I)\cdot\frac{\partial F(I,t)}{\partial I}\right)=0$$

Full profile



- The current solution ignores diffusion. A complete solution, which take into account both cooling and diffusion, is needed.
- An analytic solution is valuable in understanding the physical process, benchmarking codes and providing informations to diagnostics.

Evolution of the ion beam under cooling II: Macro-particle tracking



Many subroutines follow the stochastic cooling code written by M. Blaskiewicz.

Current subroutine for CeC one turn kick:

$$E_{1D}\left(\tilde{z}\right) \approx E_p e^{-\frac{\tilde{z}^2}{2\sigma_{z,rms}^2}} \sin\left(k_0 \tilde{z} - \varphi_0\right)$$

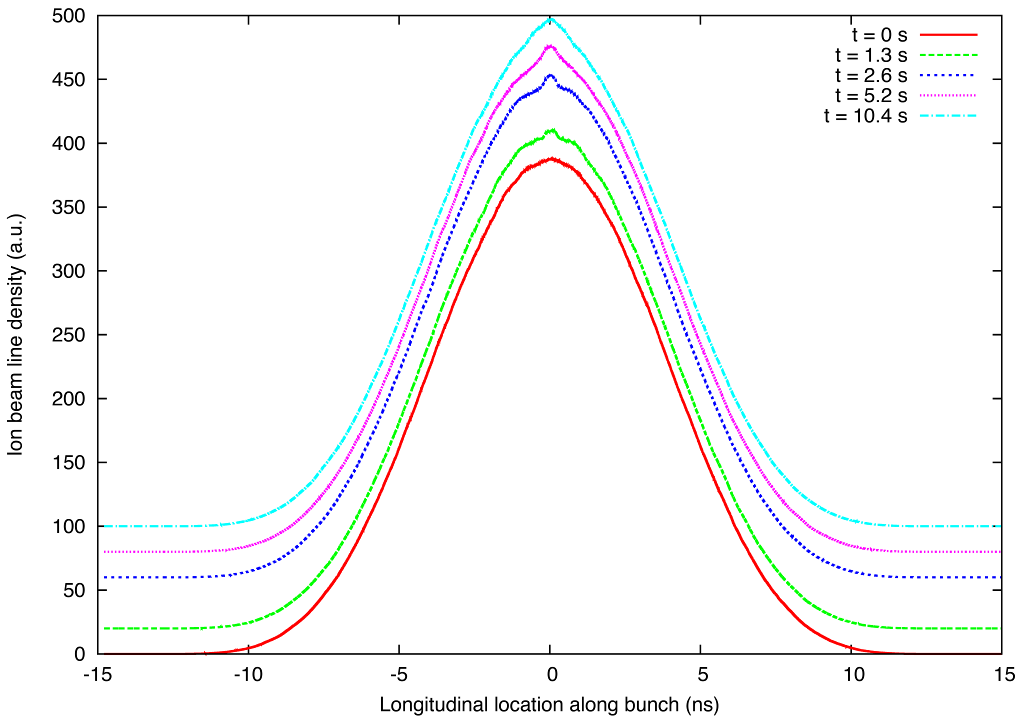
$$\Delta E_{j,N} \approx -Z_i e E_p l_1 \sin\left(k_0 D \cdot \delta_j\right) + Z_i e E_p l_1 \sqrt{\frac{3}{2}\sqrt{\pi}\rho_{ion}\left(\varsigma_j\right)\sigma_{z,rms}} \cdot X_{j,N} + e E_p l_1 \sqrt{\frac{3}{2}\sqrt{\pi}\rho_e \sigma_{z,rms}} \cdot Y_{j,N}$$

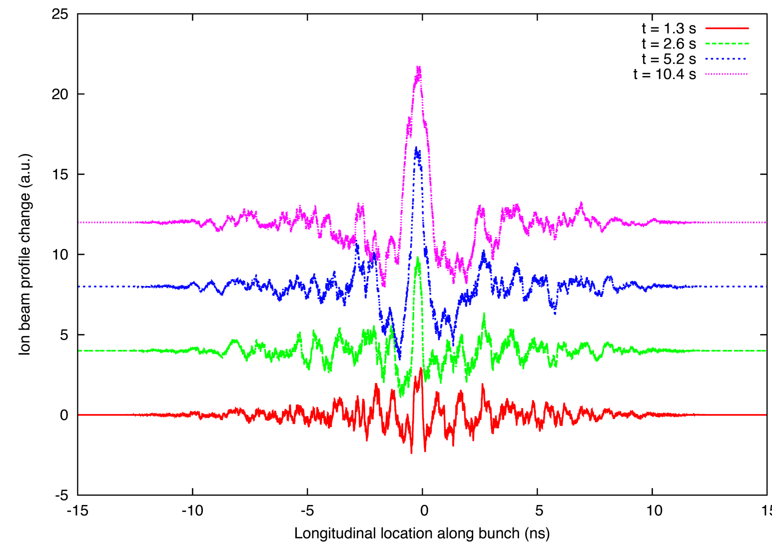
$$\times R \qquad\qquad \times\sqrt{R} \qquad\qquad \times\sqrt{R}$$

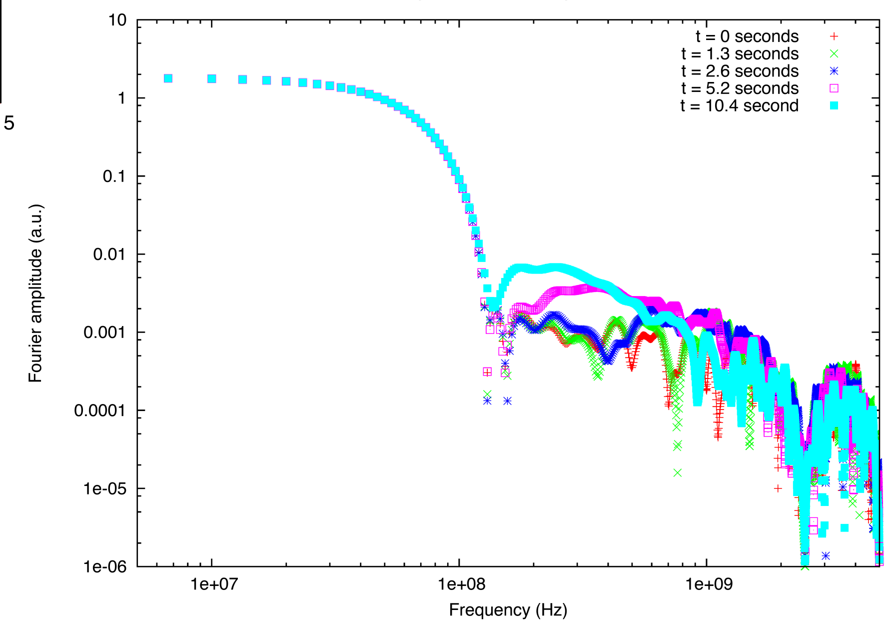
The code also use the scaling law to reduce simulation time, i.e. multiplying the coherent cooling kick by R and the incoherent diffusive kick (due to CeC and IBS) by $R^{1/2}$.

$$R = \frac{\text{Number of turns in real machine}}{\text{Number of turns in simulation}}$$

Simulation of ion beam I:
Ion beam profile evolution in 10 seconds (preliminary)







Electron bunch charge: 3 nC

Electron bunch length: 30 ps

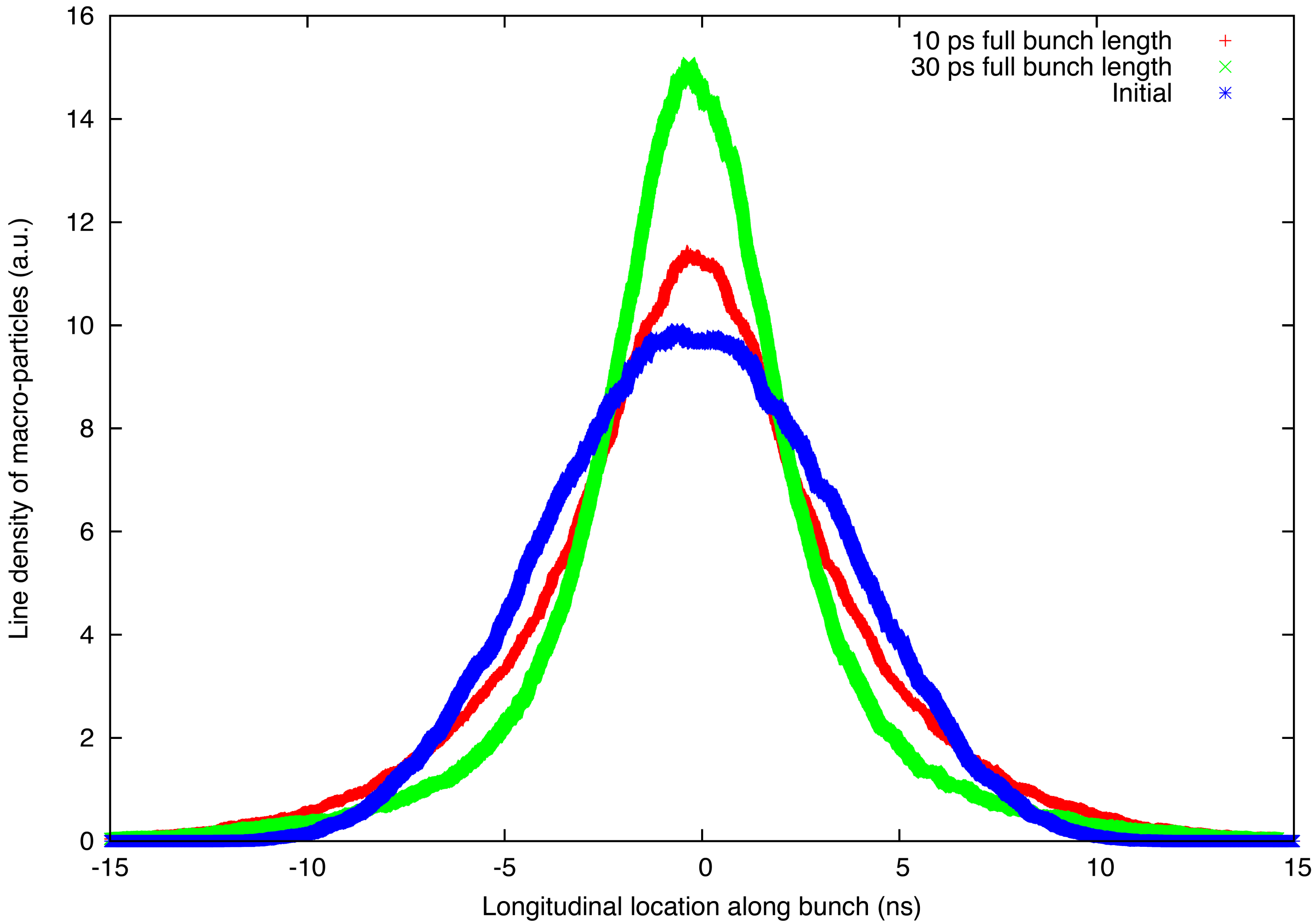
Ion bunch intensity: 1.6E8

Ion bunch length: 3.5 ns

Ion energy spread (RMS): 3.8 E-4

CeC gain: 100 (E_p=50 V/m)

Simulation of ion beam II:
Ion beam profile after 40 minutes (preliminary)



Summary

What has been worked on...

- Analytical approach to model the three sections of CeC has been explored with uniform spatial distribution. The results from the analytical studies have been used to benchmark numerical simulation and to understand the relevant physics process.
- 1-D FEL theory is used to obtain the wave-packet after amplification. In high-gain limit, the wave-packet can be approximated with a Gaussian envelop which is currently applied in the cooling simulation (i.e. tracking ions under cooling).
- Simulation of modulator and kicker section has bee done for uniform (infinite) beam profile which agrees with the analytical solution. For finite beam, the simulation is still work in progress.
- We are using Genesis to simulate the FEL amplifier which shows that the gain of 100 can be achieved without being saturated by the electron shot noise.
- A tracking code is currently under development to simulate ion beam under CeC. The preliminary results show expected local 'peak' as well as long-term cooling of the ion beam.

What is on the to do list ...

- Continue developing and improving the tracking code.
- Developing analytic solution to predict evolution of ion beam under cooling.
- Feasibility studies of adjusting the gain of FEL amplifier via phase shifter;
- Proceeding on modeling CeC for a finite electron beam

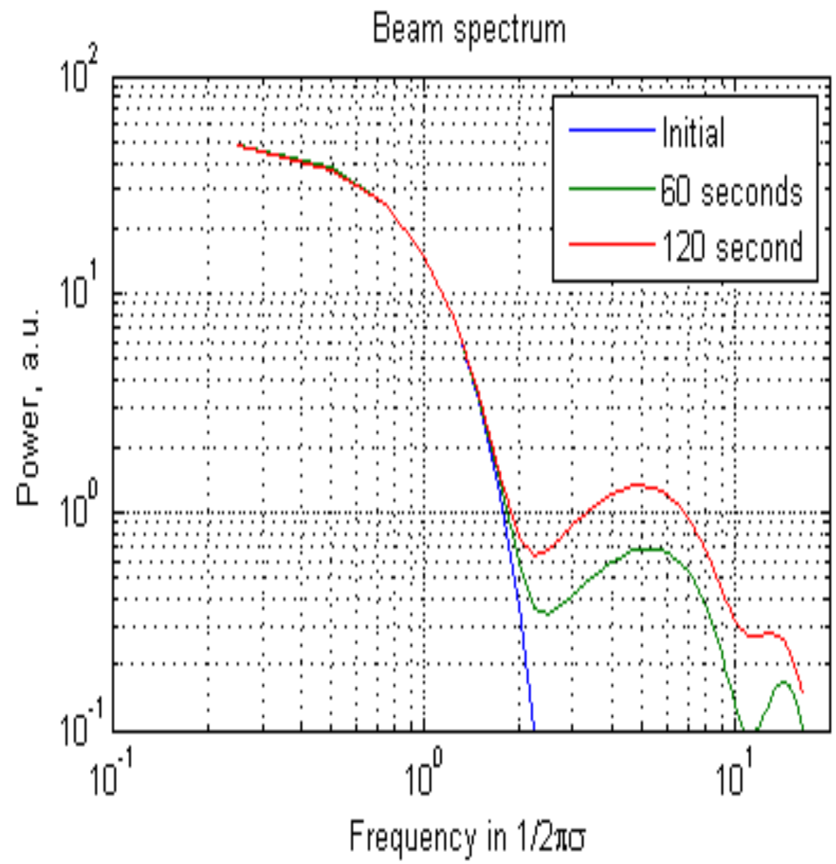
...

Backup slides

Goals of CeC APEX in run 12 con. 2

– Ion bunch spectrum

- ✓ The longitudinal size of the 'bump' due to local cooling is determined by electron bunch size.
- ✓ Betacool simulation suggests that an increase of the spectrum power in the range from 200 MHz to 800 MHz should be observed after 60 seconds of effective cooling, if the noise level allow.
- ✓ Thus, it is essential to measure the ion beam spectrum and noise level in the relevant frequency range.



Numerical solution of the Vlasov-Poisson system for the finite beam.

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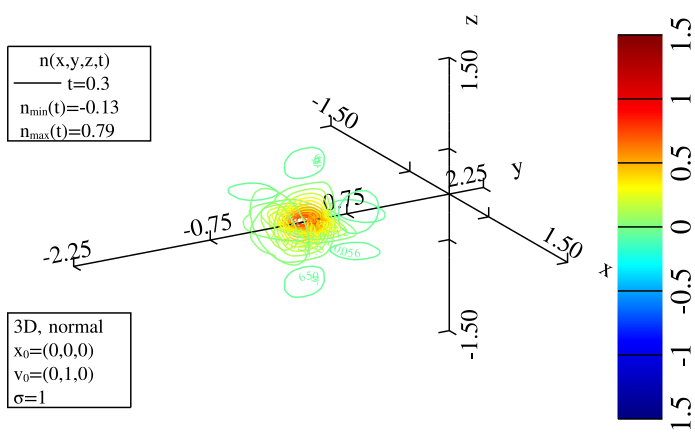
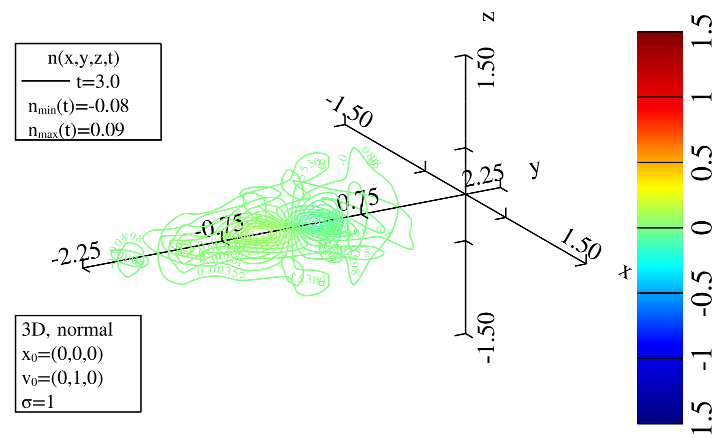
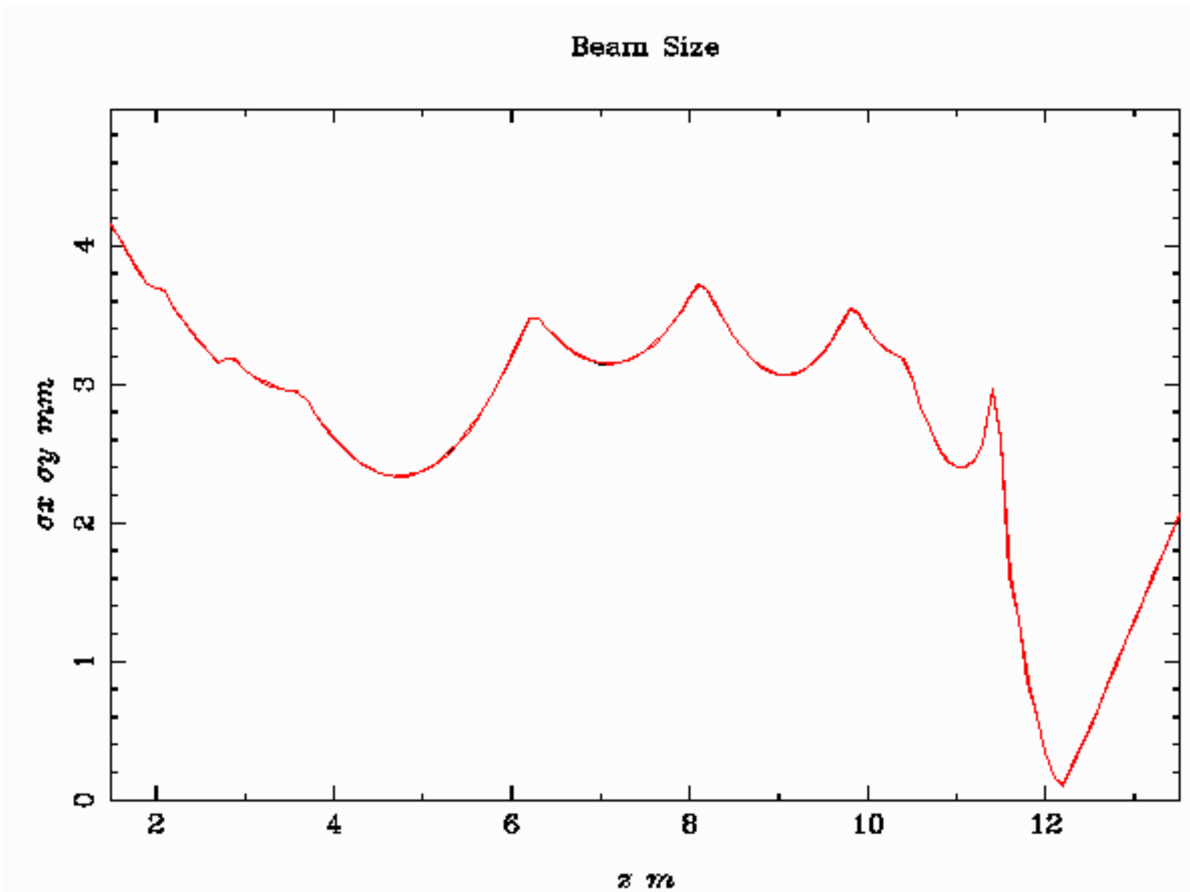
- It's very important to be able to model the modulator for the realistic finite beam.
- We developed a numerical solver for the Vlasov-Poisson system for the finite beam with confining fields. The solver was tested on the exactly solvable equations (non-physical).
- We obtained numerical results for the 1D, 2D and 3D balls with the normal velocity and spatial distributions. Below we present the results for the 3D case:





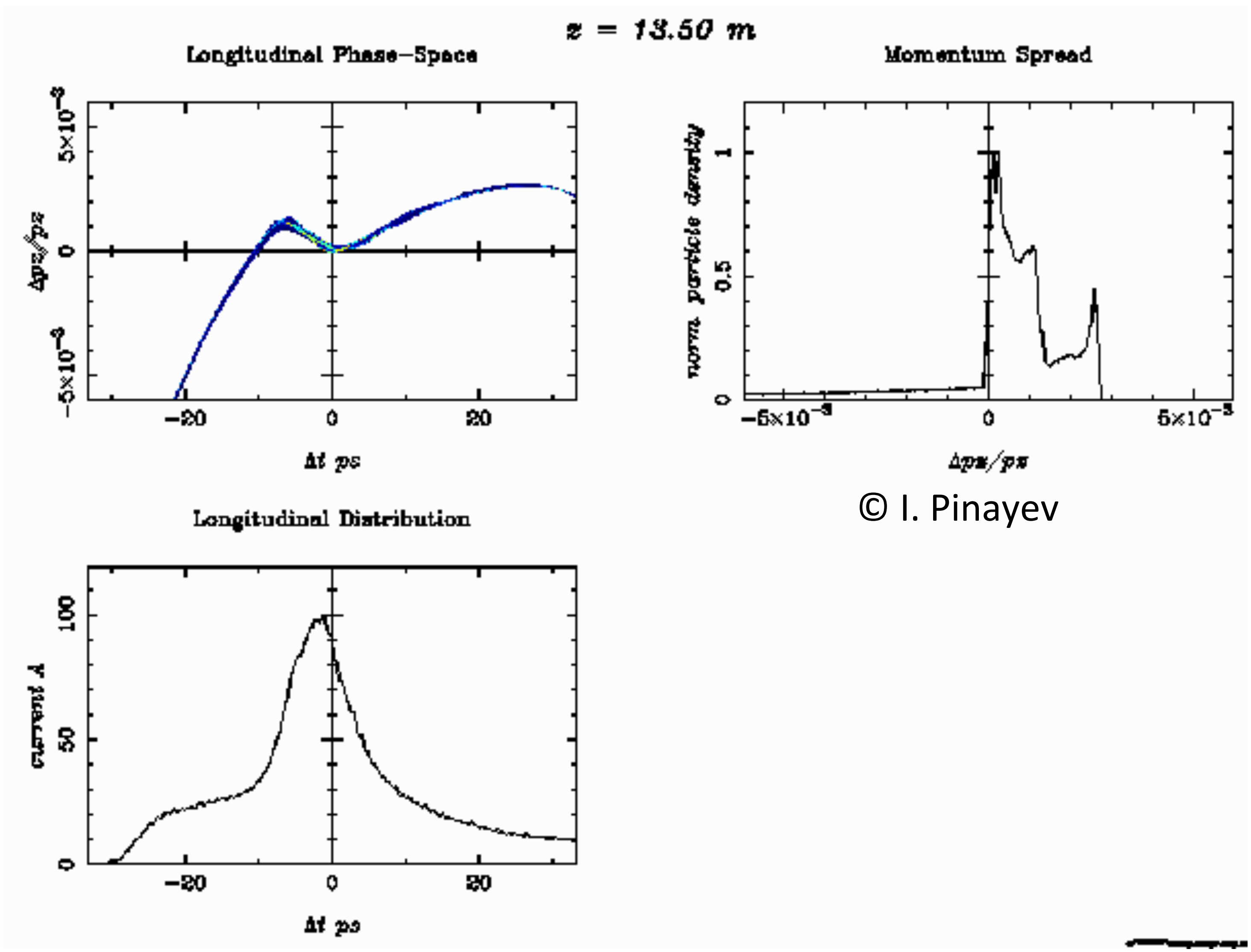
The finite plasma results have some distinctive qualitative features:

- Due to finiteness and conservation of the total charge, the perturbation is always accompanied by the negative peak.
- The plasma waves are reflected for the plasma's effective boundary.
- There are oscillations of the shape of the perturbations in the confining fields.



Beam Size
σx σy mm
z m
0
1
2
3
4
2
4
6
8
10
12

Electron beam dynamics simulation II



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